

**KUNCI JAWABAN UJI KOMPETENSI AKHIR SEMESTER 1
MATEMATIKA X**

I. Pilihan Ganda

1. **Jawaban: d.** $a^{20}b^{10}$

Pembahasan:

$$\frac{a^{-\frac{4}{5}} \left(a^{\frac{4}{5}} b^{\frac{1}{4}} \right)^2}{b^{-\frac{1}{5}} \frac{a^{\frac{1}{4}}}{b^{\frac{1}{5}}}} = a^{20} b^{10}$$

2. **Jawaban: b.** $\frac{2y^7}{3x^4}$

Pembahasan:

$$\frac{\frac{2}{3} (x^{-3} y^5)^2}{\frac{x^{-2}}{y^{-3}}} = \frac{2y^7}{3x^4}$$

3. **Jawaban: c.** $\frac{p^{\frac{7}{6}}}{\sqrt{p^{\frac{19}{3}}}}$

Pembahasan:

$$\frac{\sqrt[3]{p^3} \sqrt[3]{p} \sqrt{p}}{\sqrt{p^6} \sqrt[3]{p}} = \frac{p^{\frac{7}{6}}}{\sqrt{p^{\frac{19}{3}}}}$$

4. **Jawaban: c.** 9

Pembahasan:

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} + \frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}-\sqrt{7}} = \frac{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7}) + (\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})}{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})} = 9$$

5. **Jawaban: b.** $\frac{38}{27}$

Pembahasan:

$$\frac{{}^6\log 216 + {}^{64}\log 2}{{}^2\log 4 + {}^{16}\log 2} = \frac{38}{27}$$

6. **Jawaban: b.** $x + 2$

Pembahasan:

$${}^5\log 3 = x \rightarrow {}^5\log 75 = {}^5\log 3 + {}^5\log 25 \\ = x + 2$$

7. **Jawaban: e.** 0

Pembahasan:

$$\log x^3 + \log \frac{1}{x} - 2\log x = 3\log x - \log x - 2\log x = 0$$

8. **Jawaban: a.** $\frac{2a-2}{2a+1}$

Pembahasan:

$$ax^2 = ax - 1 \rightarrow ax^2 - ax + 1 = 0$$

$$\frac{x_2}{x_1+1} + \frac{x_1}{x_2+1} = \frac{x_2^2 + x_2 + x_1^2 + x_1}{x_1x_2 + x_1 + x_2 + 1}$$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) + \left(-\frac{b}{a}\right)}{\frac{c}{a} + \left(-\frac{b}{a}\right) + 1}$$

$$= \frac{1 - \frac{2}{a} + 1}{\frac{1}{a} + 1 + 1} = \frac{2 - \frac{2}{a}}{2 + \frac{1}{a}}$$

$$= \frac{\frac{2a-2}{a}}{\frac{2a+1}{a}} = \frac{2a-2}{2a+1}$$

9. **Jawaban: c. 11**

Pembahasan:

$$x^2 - px + 24 = 0$$

$$x_1 - x_2 = 5 \rightarrow \frac{\sqrt{D}}{a} = 5$$

$$\frac{\sqrt{p^2 - 96}}{1} = 5$$

$$p^2 - 96 = 25$$

$$p^2 = 121$$

$$p = \sqrt{121} = 11$$

10. **Jawaban: d.** $p \leq 3$

Pembahasan:

$$x^2 + 6x + (2p + 3) \rightarrow \text{akarnya berlainan dan riil}$$

$$D > 0 \rightarrow 36 - 4 \cdot 1(2p + 3) > 0$$

$$36 - 8p - 12 > 0$$

$$-8p > -24$$

$$p \leq 3$$

11. **Jawaban: b.** $\{(1, -2)\}$

Pembahasan:

$$3x + y = 1 \quad | \times 3 | \quad 9x + 3y = 3$$

$$2x - 3y = 8 \quad | \times 4 | \quad 8x - 12y = 32$$

$$11x = 11 \rightarrow x = 1$$

$$x = 1 \rightarrow 3 + y = 1 \rightarrow y = -2$$

$$H_p = \{(1, -2)\}$$

12. **Jawaban: c. 50 dan 10**

Pembahasan:

$$a + b = 60$$

$$\frac{a}{2} = \frac{5b}{2} \rightarrow a = 5b$$

$$a = 5b \rightarrow 5b + b = 60$$

$$6b = 60 \rightarrow b = 10$$

$$b = 10 \rightarrow a = 50$$

bilangan-bilangan itu adalah 50 dan 10

13. **Jawaban:** a. $\{(-1, 0), (1, 0)\}$

Pembahasan:

$$x^2 - 1 = 1 - x^2$$

$$2x^2 = 2 \rightarrow x^2 = 1 \rightarrow x = \pm 1$$

$$\left\{ \begin{array}{l} x = -1 \rightarrow y = 0 \\ x = 1 \rightarrow y = 0 \end{array} \right\} \{(-1, 0), (1, 0)\}$$

14. **Jawaban:** c. $\{x \mid 2 < x < 6\}$

Pembahasan:

$$\sqrt{x^2 + x - 6} < x$$

$$x^2 + x - 6 \geq 0 \rightarrow (x + 3)(x - 2) \geq 0$$

$$x = -3 \text{ atau } x = 2$$

$$x^2 + x - 6 < x^2$$

$$x - 6 < 0 \rightarrow x < 6$$

$$\text{Hp}\{x \mid 2 < x < 6\}$$

15. **Jawaban:** a. $\{x \mid 2 \leq x \leq 5\}$

Pembahasan:

$$\frac{1-2x}{2-x} \geq 3$$

$$\frac{1-2x-6+3x}{2-x} \geq 0$$

$$\frac{x-5}{2-x} \geq 0$$

$$x = 5 \text{ atau } x = 2$$

$$\text{Hp} = \{x \mid 2 \leq x \leq 5\}$$

16. **Jawaban:** d. $\{x \mid -\frac{11}{3} \leq x \leq 3\}$

Pembahasan:

$$(3x+1) \leq 10$$

$$3x + 4 \geq -10 \text{ atau } 3x + 1 \leq 10$$

$$3x \geq -11 \quad 3x \leq 9$$

$$x \geq -\frac{11}{3} \quad x \leq 3$$

$$\text{Hp} = \{x \mid -\frac{11}{3} \leq x \leq 3\}$$

17. **Jawaban:** e. 2

Pembahasan:

$$(\sqrt{2})^{x-2} = \frac{1}{8^{x-2}}$$

$$2^{\frac{x-2}{2}} = 2^{-3x+6}$$

$$\frac{x-2}{2} = -3x+6$$

$$x-2 = -6x+12$$

$$7x = 14 \rightarrow x = 2$$

18. **Jawaban:** d. $-3 < x < 1$ atau $1 < x < 5$

Pembahasan:

$$\frac{1}{2} \log(x^2 - 2x = 1) > -4$$

$$\frac{1}{2} \log(x-1)^2 > {}^2\log 2^{-4}$$

$$\frac{1}{(x-2)^2} > \frac{1}{16}$$

$$\text{-) } x-1 \geq 0$$

$$x \geq 1$$

$$\text{-) } x^2 - 2x - 15 < 0$$

$$(x+3)(x-5) < 0$$

$$x = -3 \text{ atau } x = 5$$

$$Hp = \{x \mid 1 < x < 5\} \text{ atau } \{x \mid -3 < x < 1\}$$

19. **Jawaban:** e. $x < -2$ atau $x > 2$

Pembahasan:

$$(x^2 + 2)^2 - 5(x^2 + 2) > 6$$

$$\text{Misal } (x^2 + 2) = a$$

$$a^2 - 5a - 6 > 0$$

$$(a+1)(a-6) > 0$$

$$a = 1 \text{ atau } a = 6$$

$$a = 1 \rightarrow x^2 + 2 = 1$$

$$x^2 = -1$$

$$x =$$

$$a = 6 \rightarrow x^2 + 2 = 6$$

$$x^2 = 4$$

$$x = \pm 2$$

$$Hp = \{x < -2 \text{ atau } x > 2\}$$

20. **Jawaban:** b. $-\frac{4}{3} < x \leq 4$

Pembahasan:

$$\sqrt{3x+4} > x$$

$$\text{-) } 3x+4 \geq 0$$

$$3x \geq -4$$

$$x \geq -\frac{4}{3}$$

$$\text{-) } 3x+4 > x^2$$

$$x^2 - 3x - 4 < 0$$

$$(x+1)(x-4) < 0$$

$$x = -1 \text{ atau } x = 4$$

$$\{x \mid -\frac{4}{3} \leq x \leq 4\}$$

21. **Jawaban:** a. $x \geq 3$

Pembahasan:

$$\sqrt{3x+9} > \sqrt{2x-6}$$

$$\begin{aligned} -) 3x - 9 &\geq 0 & -) 2x - 6 &\geq 0 \\ X &\geq 3 & 2x &\geq 6 \rightarrow \geq 3 \\ -) 3x - 9 &> 2x - 6 \\ x &> -15 \end{aligned}$$

$$\{x \mid x \geq 3\}$$

22. **Jawaban: d.** $1 < x \leq 2$

Pembahasan:

$$\frac{1}{x-1} \geq 1 \rightarrow x-1 \geq 0$$

$$\begin{aligned} x-1 &\leq 1 & x &\geq 1 \\ x &\leq 2 \end{aligned}$$

$$\{x \mid 1 < x \leq 2\}$$

23. **Jawaban: a. 5**

Pembahasan:

$$x^2 - 4x + (p+1) = 0$$

Syarat memiliki dua akar kembar $D = 0$.

$$16 - 4p + 4 = 0$$

$$-4p = -20$$

$$p = 5$$

24. **Jawaban: e. $f(x) = 2x^2 - 4x + 4$**

Pembahasan:

Titik puncak (1, 2) melalui titik (2, 4)

➤ Fungsi kuadrat $ax^2 + bx + c = y$

$$(1, 2) \rightarrow a + b + c = 2$$

$$(2, 4) \rightarrow 4a + 2b + c = 4$$

➤ Titik puncak $\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

$$-\frac{b}{2a} = 1 \rightarrow b = -2a$$

➤ $a + (-2a) + c = 2 \rightarrow -1 + c = 2$

$$4a - 4a + c + 4 \rightarrow c = 4$$

$$-a = -2$$

$$a = 2$$

➤ $a = 2, c = 4$

$$a + b + c = 2 \rightarrow 2 + b + 4 = 2$$

$$b = 2 - 6 = -4$$

Fungsi kuadratnya: $f(x) = 2x^2 - 4x + 4$.

25. **Jawaban: e. 10 dan 7**

Pembahasan:

Misalkan kedua bilangan tersebut a dan b

$$2a + 3b = 41$$

$$4a - 3b = 19 +$$

$$6a = 60$$

$$a = 10$$

Nilai $a = 10$ disubstitusikan ke persamaan $2a + 3b = 41$

$$2 \cdot 10 + 3b = 41$$

$$20 + 3b = 41$$

$$3b = 41 - 20$$

$$3b = 21$$

$$b = 7$$

Jadi, $a = 10$ dan $b = 7$.

II. Uraian

1. Jawab:

$$\begin{aligned} \text{a. } & (3 - \sqrt{5})\sqrt{3 - \sqrt{5}} + (3 + \sqrt{5})\sqrt{3 - \sqrt{5}} \\ &= (3 - \sqrt{5})(3 + \sqrt{5}) \left(\frac{1}{\sqrt{3 + \sqrt{5}}} + \frac{1}{\sqrt{3 - \sqrt{5}}} \right) \\ &= 9 - 5 \left(\frac{\sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}}{2} \right) \\ &= 2(\sqrt{3 - \sqrt{5}} + \sqrt{3 + \sqrt{5}}) \end{aligned}$$

$$\begin{aligned} \text{b. } & \sqrt{7 + \sqrt{40}} = \sqrt{7 + 2\sqrt{10}} \\ &= \sqrt{(\sqrt{5} + \sqrt{2})^2} \\ &= \sqrt{5} + \sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{c. } & \frac{2}{-3 + \sqrt{3}} + \frac{2}{-3 + 2\sqrt{3}} = \frac{2(2 + 2\sqrt{3}) + 3(\sqrt{3} - 3)}{(\sqrt{3} - 3)(2 + 2\sqrt{3})} \\ &= \frac{4 + 4\sqrt{3} + 3\sqrt{3} - 9}{2\sqrt{3} + 6 - 6 - 6\sqrt{3}} \\ &= \frac{7\sqrt{3} - 5}{-4\sqrt{3}} \\ &= \frac{5\sqrt{3} - 21}{12} \end{aligned}$$

2. Jawab:

- a. $x \leq \frac{2}{3}$ atau $x \geq 3$
b. $2 \leq x \leq 3$

3. Jawab:

Misalkan x mewakili bilangan pertama maka bilangan kedua sama dengan $2x$. Jumlah kedua bilangan itu kurang dari 50, sehingga model matematikanya adalah

$$\begin{aligned} x + 2x &< 50 \\ \Leftrightarrow 3x &< 50 \end{aligned}$$

Penyelesaian model matematika tersebut adalah $3x < 50 \Leftrightarrow x < 16\frac{2}{3}$. Jadi batas bilangan pertama tidak lebih dari $16\frac{2}{3}$, sedangkan bilangan kedua tidak lebih dari $33\frac{1}{3}$.

4. Jawab:

a. $\sqrt{x^2 + x + 4} \leq 4$

$$x^2 + x + 4 \leq 16$$

$$x^2 + x - 12 \leq 0$$

$$(x + 4)(x - 3) \leq 0$$

$$x = -4 \text{ atau } x = 3$$

$$\text{Hp} = \{ -4 \leq x \leq 3 \}$$

b. $|x-1| + |2x-5| < 3$

$$x - 1 + 2x - 5 > -3 \text{ atau } x - 1 + 2x - 5 < 3$$

$$3x > 3$$

$$3x < 9$$

$$x > 1$$

$$x < 3$$

$$\text{Hp} = \{ 1 < x < 3 \}$$

5. **Jawab:**

a.
$$\left. \begin{aligned} y &= 2x^2 - 3x + 6 \\ y &= 3x^2 - 10x + 18 \end{aligned} \right\}$$

$$2x^2 - 3x + 6 = 3x^2 - 10x + 18$$

$$3x^2 - 2x^2 - 10x + 18 - 6 + 3x = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x = 4 \text{ atau } x = 3$$

$$x = 4, \text{ maka } y = 2(4)^2 - 3(4) + 6 = 32 - 12 + 6 = 26$$

$$x = 3, \text{ maka } y = 2(3)^2 - 3(3) + 6 = 18 - 9 + 6 = 15$$

$$(4, 26) \text{ dan } (3, 15)$$

b.
$$\left. \begin{aligned} y &= 2x^2 - 10x - 30 \\ y &= 6x^2 - 38x + 18 \end{aligned} \right\}$$

$$2x^2 - 10x - 30 = 6x^2 - 38x + 18$$

$$6x^2 - 2x^2 - 38x + 18 + 30 + 10x = 0$$

$$4x^2 - 28x + 48 = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x - 4)(x - 3) = 0$$

$$x = 4 \text{ atau } x = 3$$

$$x = 4, \text{ maka } y = 2(4)^2 - 10(4) - 30 = 32 - 40 - 30 = -38$$

$$x = 3, \text{ maka } y = 2(3)^2 - 10(3) - 30 = 18 - 30 - 30 = -42$$